MATH 141 | HOMEWORK 4 | DUE DATE: THURSDAY, MAY 25TH

Name: Solutions

> This homework is due Thursday, May 25th during recitation. If you have questions regarding any of this, feel free to ask during office hours or send me an email. When writing solutions, present your answers clearly and neatly, showing only necessary work.

1. Find the equation of the tangent line to the curve satisfying $xy + 2x - 5y = 2$ at the point $(3, 2)$.

$$x\frac{dy}{dx} + y + 2 - 5\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y+2}{5-x} \quad @\,(3,2) = \frac{2+2}{5-3} = 2$$

$$y - 2 = 2(x-3)$$

Answer: $y = 2x - 4$

2. Coffee is draining from a conical filter, with height and diameter 6cm, into a cylindrical coffee pot, also of diameter 6cm, at a rate of $10\text{cm}^3/\text{min}$.

   (a) How fast is the level in the pot rising when the coffee in the cone is $5cm$ deep.

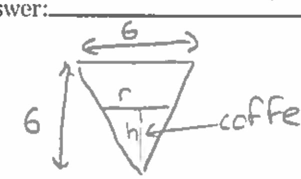
Diameter = 6
Radius = 3

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{\pi r^2}\frac{dV}{dt} = \frac{1}{\pi 3^2}\cdot 10$$

Answer: $\frac{10}{9\pi}$ cm/min

   (b) How fast is the level in the cone falling at that time?



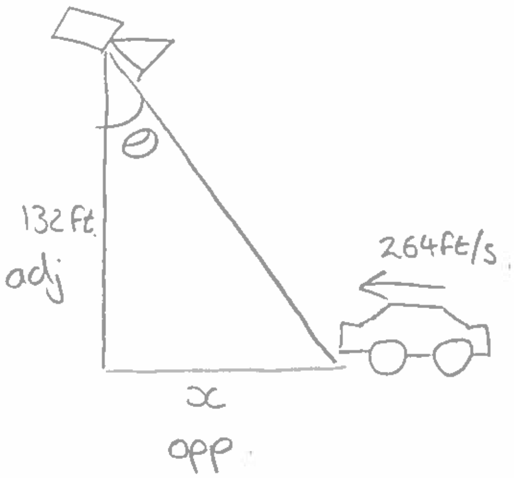
$$\frac{r}{h} = \frac{3}{6} \Rightarrow r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12}\cdot 3h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4}{h^2\pi}\frac{dV}{dt} = \frac{4}{5^2\cdot\pi}(-10)$$

Answer: $-\frac{8}{5\pi}$ cm/min

3. You are videotaping a race from a stand 132ft from the track, following a car that is moving at 264ft/sec. How fast will the angle of the camera be changing when the car is right in front of you? (See question 38 on page 201 for a picture)



$\theta = 0$

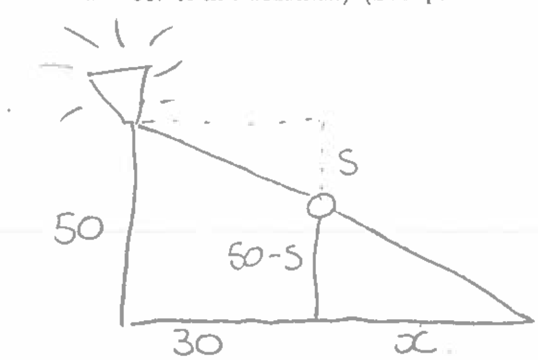
$$\tan(\theta) = \frac{x}{132}$$

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{132}\frac{dx}{dt}$$

$$\underset{1}{\sec^2(0)}\frac{d\theta}{dt} = \frac{1}{132}(-264)$$

Answer: -2 radians/sec

4. A light shines from the top of a pole 50ft high. A ball is dropped from the same height from a point 30ft away from the light. How fast is the shadow of the ball moving along the ground 0.5 seconds later? (assume the ball falls a distance $s = 16t^2$ft in t seconds.) (See question 39 on page 201 for a picture)



$$\frac{50-s}{x} = \frac{50}{x+30}$$

$$(50-s)(x+30) = 50x$$

$$-sx + 1500 - 30s = 0$$

$$sx = 1500 - 30s$$

$$16t^2 x = 1500 - 480t^2$$

$$@\, t \neq 0,\ t^2 \neq 0 \implies x = \frac{1500}{16}t^{-2} - 480$$

$$\frac{dx}{dt} = \frac{-3000}{16t^3}$$

$$@\, t = 1/2 \qquad = -\frac{3000}{16(1/2)^3}$$

Answer: -1500 ft/s

Cont.

5. Find the absolute maximum and minimum values of each function on the given interval:

(a) $f(x) = \frac{2}{3}x - 5,\ -2 \leq x \leq 3$

$f'(x) = \frac{2}{3} \implies$ no critical points

$f(-2) = \frac{2}{3}(-2) - 5 = \frac{-19}{3}$

$f(3) = \frac{2}{3}(3) - 5 = -3 = \frac{-9}{3}$

Answer: Abs max $= -3$ Abs min $= -\frac{19}{3}$

(b) $f(x) = 4 - x^3,\ -2 \leq x \leq 1$

$f'(x) = -3x^2 \implies 0$ is a critical point

$f(-2) = 4 - (-2)^3 = 4 + 8 = 12$

$f(0) = 4 - 0^3 = 4$

$f(1) = 4 - 1^3 = 3$

Answer: Abs max $= 12$ Abs min $= 3$

(c) $f(x) = x^2 - 1,\ -1 \leq x \leq 2$

$f'(x) = 2x \implies 0$ is a critical point

$f(-1) = (-1)^2 - 1 = 0$

$f(0) = 0^2 - 1 = -1$

$f(2) = 2^2 - 1 = 3$

Answer: Abs max $= 3$ Abs min $= -1$